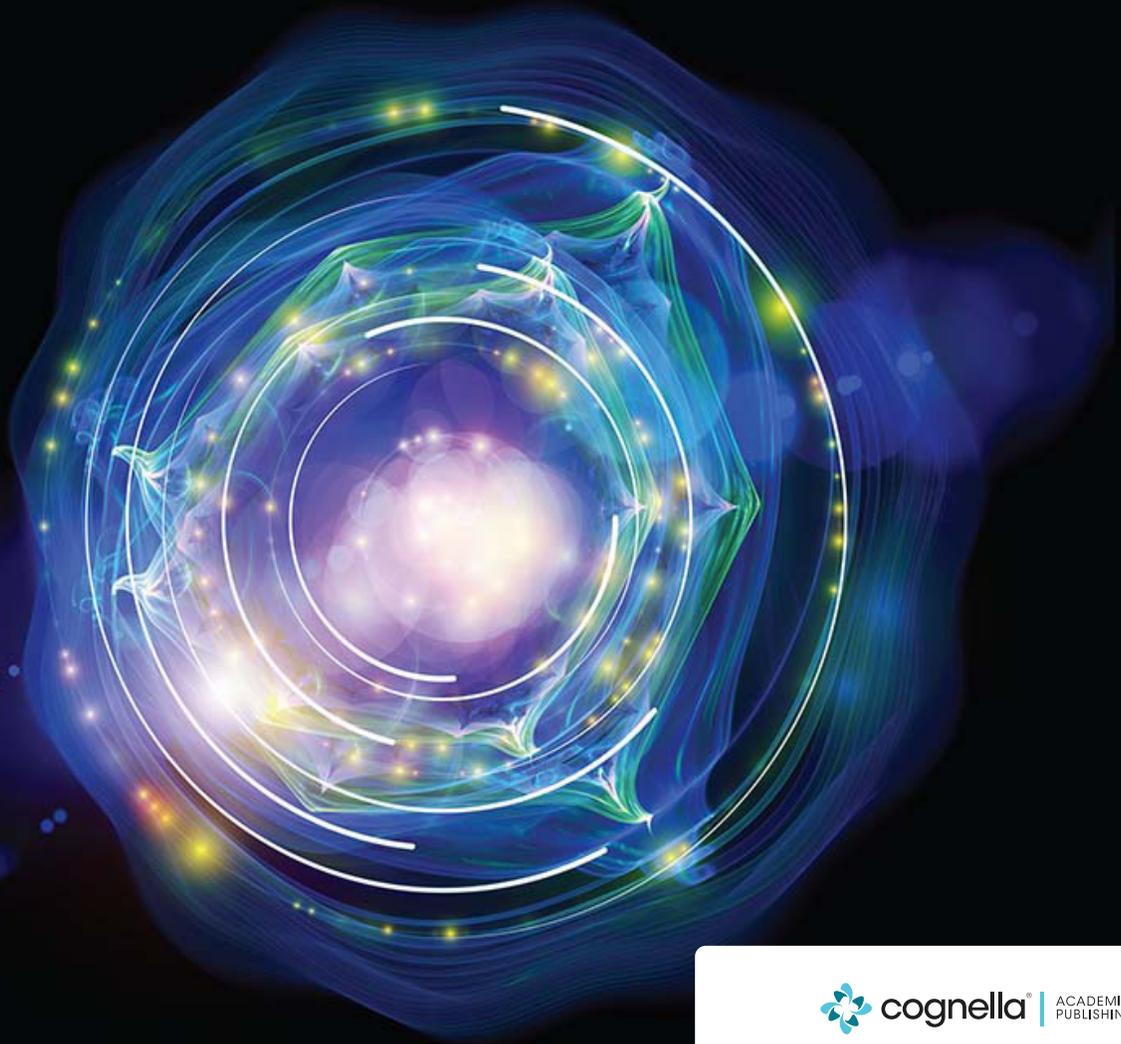


INTERMEDIATE ALGEBRA

The Why and the How



Mathew Baxter

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INTERMEDIATE

ALGEBRA

The Why and the How

First Edition

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CHAPTER

1

The Real Number System

1.1: REAL NUMBERS AND THEIR PROPERTIES

We begin our studies with the definitions of the kinds of numbers we will be using in this text: real numbers. We also discuss the important properties of real numbers.

Real Numbers

First, we'll discuss sets in general, so that we have good terminology when we move on to sets of numbers.

Sets are groups of objects, called elements. When we list the elements of a set, we usually make a comma-separated list surrounded by braces.

Ex) The set of days has seven elements: the five weekdays as well as Saturday and Sunday. If we call the set of days D for short, then

$$D = \{\text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}\}.$$

Ex) When we flip a fair coin, the set of possibilities consists of two elements: heads and tails. In statistics, this set of outcomes is called the **sample space**.

We need to be able to tell if an element actually belongs to a set. We use the raised Greek letter *epsilon*, ϵ , to mean "is an element of."

Ex) If we call the set of days D , then Monday is an element of the set D . We can write this as Monday ϵD .

Ex) In our coin example, if we call the outcome of heads H and we call the set of all outcomes when flipping a fair coin Ω (a capital omega), we can write $H \in \Omega$. In this example, Ω consists of "heads" or "tails," H or T for short. So we can say $\Omega = \{H, T\}$.

Sets can be infinite. We use an ellipsis (a trail of periods, like "...") to indicate that a set continues. For example, the set of positive even numbers $\{2, 4, 6, 8, \dots\}$ is an infinite set.

In this text, the most important sets will be sets of numbers. See Figure 1-1.

The natural numbers are the counting numbers 1, 2, 3,

The natural numbers are denoted by \mathbb{N} . So we may write $\mathbb{N} = \{1, 2, 3, \dots\}$.

The whole numbers are the natural numbers, including 0. So the whole numbers are 0, 1, 2, The set of whole numbers will not be used much in this text. In Figure 1-1, whole numbers would be between natural numbers and integers.

The integers are the natural numbers, their opposites, and zero.

Recall that the opposite of a number is the same number across from zero on a number line. So the opposite of 2 is -2 , and the opposite of -3 is 3. See Figure 1-2.

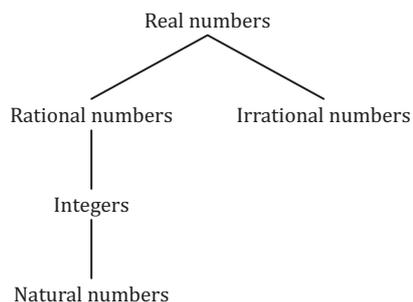


Fig. 1-1
Types of Numbers

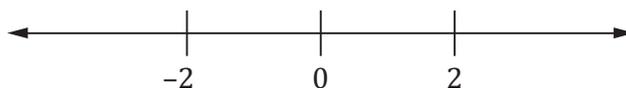


Fig. 1-2
2 is the opposite of -2

The integers are denoted by \mathbb{Z} . The “Z” stands for *zahlen*, which is the German word for *integer*. So we can write $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$.

Thus, the integers are all of the usual tick marks when we make a number line. See Figure 1-3.

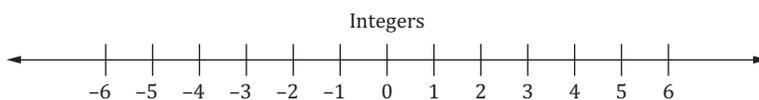


Fig. 1-3
Integers at the lattice points

Ex) The number -7 is an integer, and the number 14 is an integer. But of the two of them, only 14 is a natural number because it is positive.

Definition: The rational numbers (or rationals) are the set of all numbers $\frac{a}{b}$ where a and b are both integers and b cannot be zero.

Sometimes we need to describe specific elements of a set. These elements have a property that differentiates them from other elements and are identified using a colon or a vertical bar.

Ex) If we want the variable x to have the property that x is an integer, then we write $x \in \mathbb{Z}$. If we want to describe all positive integers, then we would write $\{y \in \mathbb{Z} : y > 0\}$ or $\{y \in \mathbb{Z} | y > 0\}$. Think about it: is there another name for this set?

Ex) Suppose we wanted to describe the integers greater than 5 . We can write this as $\{a \in \mathbb{Z} : a > 5\}$. First, we are describing a general statement, saying that we want our arbitrary number a to be an integer. Then, after the “such that” colon, we are describing a more specific trait that we want a to have.

YOU TRY IT: Using set notation, describe all of the integers less than -2 .

Ex) Let's describe the integers that are greater than 5 and are also even. We can separate the requirements on our arbitrary integer a using commas, so we could write this set as $\{a \in \mathbb{Z} : a > 5, a \text{ even}\}$.

The rationals are denoted by \mathbb{Q} . Look again at the definition and you will see why the letter “Q” was chosen. It stands for “quotients” (of integers). We can write the definition of the rational numbers in set notation as

$$\mathbb{Q} = \left\{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0 \right\}.$$

Ex) The number $\frac{4}{7}$ is rational.

Ex) Anything below the rational numbers in Figure 1-1 is also a rational number. For example, -4 is a rational number, because we can write it as the quotient of -4 and 1 : $\frac{-4}{1}$. And 18 is a rational number, because $18 = \frac{18}{1}$.

Ex) The number $0.333 \dots$ is rational, because this is the decimal representation of $\frac{1}{3}$. The number $0.1\overline{6}$ is rational, because this is the decimal representation of $\frac{1}{6}$.

What's important to remember about the previous example is that *repeating decimals can be represented as fractions*, so they are rational numbers. This is one key distinction between rational numbers and irrational numbers.

Any real number that is not rational is *irrational*. For example, $\sqrt{2}$ is irrational because it cannot be written as the quotient of integers. Another example is π , because $\pi = 3.141592653 \dots$ has a *nonrepeating nonterminating* decimal representation.

To clarify:

Number Type	Decimal representation	Written as a fraction?
Rational	Repeating or terminating	Can be written as a fraction
Irrational	Nonrepeating and nonterminating	Cannot be written as a fraction

Note that the decimal representation of a number must be *both* nonrepeating and nonterminating for that number to be irrational.

Ex) The decimal 3.141 is nonrepeating but it terminates, so it is a rational number. We can write it as $\frac{3,141}{1,000}$.

Ex) The decimal $-0.8888 \dots$ is nonterminating but it repeats, so it is a rational number. This is $-\frac{8}{9}$.

One final thing worth mentioning about the rationals is the restriction that the denominator may not be zero. Division by zero is not allowed. You cannot divide something into zero parts.

The real numbers make up the set that contains both the rational numbers and the irrational numbers. The notation for the real numbers is \mathbb{R} .

Example: Use the classifications of *integer*, *real number*, *rational number*, *irrational number*, and/or *natural number* to describe each of these numbers:

1. $\frac{3}{2}$
2. $\sqrt{-73}$
3. $\frac{5}{6} = 0.8333 \dots$
4. $\sqrt{7} = 2.64575 \dots$

Solution:

1. $\frac{3}{2}$ is a quotient of the integers 3 and 2, which makes it a rational number. Remember that it is also anything higher than the rationals in Figure 1-1, so it is also a real number.
2. -73 is an integer (since it is negative, it is not a natural number). Because it is an integer, it is also a rational number and a real number.
3. Note that $\frac{5}{6}$ has a repeating decimal representation, so it is a rational number. It is also a real number.
4. The difference here is that $\sqrt{7}$ has a decimal representation that is nonrepeating and nonterminating, so it is an irrational number. It is also a real number.

Properties of Real Numbers

When we add two positive real numbers together, the result is another positive real number. For example, 7 and 12 are both positive numbers, and their sum, 19, is another positive real number. If we subtract two positive real numbers, the result may not be positive. We know that $12 - 7 = 5$, and 5 is positive, but if we subtract 12 from 7 we get $7 - 12 = -5$. The real numbers are imbued with an ordering that lets us compare any two of them and decide which is larger. So when we subtract, we have to make a note about which number is larger.

If we do

larger number – smaller number ... the result is positive,

and if we do

smaller number – larger number ... the result is negative.

Ex) If we subtract 132 from 133, we end up with a positive number, because 133 is larger than 132. So $133 - 132 = 1$.

Ex) If we subtract 7 from 5, the result is a negative number, because we are subtracting a larger number from a smaller number, which is case two above. So $5 - 7 = -2$.

We can add numbers in any order, but we cannot subtract numbers in any order. We can summarize this by saying that addition of real numbers is commutative, whereas subtraction is not. We have that $a + b = b + a$, but $a - b$ is not necessarily equal to $b - a$.

Ex) $3 + 4$ and $4 + 3$ both equal 7. But $4 - 3 = 1$ while $3 - 4 = -1$.

Multiplication and division present a similar situation. We can multiply two numbers in any order, and the result is the same. But we cannot divide numbers in any order.

Ex) $3 \cdot 4 = 12$, and this is the same as $4 \cdot 3 = 12$. But note that $3 \div 4$ and $4 \div 3$ are different numbers (0.75 and $1.\bar{3}$, respectively).

Another nice property of the real numbers is that addition (and multiplication) is associative, meaning that we can add multiple numbers in any order from left to right. If we add the numbers 3, 4, and 12 from left to right, we can add 3 and 4 first or we can add 4 and 12 first. If we use parentheses to denote this, we are saying that $(3 + 4) + 12 = 3 + (4 + 12)$, where we are doing the addition in the parentheses first.

The associative property can be extended to any amount of numbers we are adding (or multiplying) together.

Ex) To add the numbers -2 , 7, 5, and 15 from left to right, we are allowed to do $(-2 + 7) + 5 + 15$, or $-2 + (7 + 5) + 15$, or $-2 + 7 + (5 + 15)$.

Another property of the real numbers is that there is an additive (and multiplicative) identity. So if you take any number, and add 0, you get the same number back. For example, $5 + 0 = 5$ and $0 + 5 = 5$. There is nothing special about the 5 here. It is the 0 that we call the additive identity.

Similarly, we can multiply a number by 1 to get the same number back. For example, $8 \cdot 1 = 1 \cdot 8 = 8$. Since we can do this for any number, not just 8, we call 1 the multiplicative identity.

Two other properties to discuss are the inverse properties, which will be extremely important when we are solving equations in the next chapter. Given any number, we would like to add something to it so we get back to the additive identity, 0. This “something” that we are adding is the additive inverse.

Ex) The additive inverse of 12 is -12 , because when we add these two numbers together we get the additive identity: $12 + (-12) = 0$, or $-12 + 12 = 0$.

Ex) The additive inverse of -2.5 is 2.5 , because $-2.5 + 2.5 = 0$.

So to find the additive inverse of a number, give its opposite.

Similarly for multiplication, the multiplicative inverse of a is the number you multiply by a to get back to the multiplicative identity 1.

Ex) The multiplicative inverse of 7 is $\frac{1}{7}$, because $7 \cdot \frac{1}{7} = \frac{7}{1} \cdot \frac{1}{7} = 1$.

Ex) The multiplicative inverse of $\frac{3}{4}$ is $\frac{4}{3}$ because $\frac{3}{4} \cdot \frac{4}{3} = 1$.

So to find the multiplicative inverse of a number, give its reciprocal.

YOU TRY IT 2: Find the sum of the additive inverse of 8 and the multiplicative inverse of 8.

The final property to discuss here is the only one that combines the operations of addition and multiplication. It is the distributive property, which tells us that $a(b + c) = ab + ac$. This is the property that allows us to distribute, where we have

$$2(x + y) = 2x + 2y,$$

but it also allows us to factor, where we can write

$$2x + 2y = 2(x + y).$$

The table below summarizes the properties of the real numbers.

YOU TRY IT ANSWERS: 1) $\{x \in \mathbb{Z} : x < -2\}$ 2) $\frac{65}{8}$

Property Name	Property Example ($a, b, c \in \mathbb{R}$)
Commutativity of Addition	$a + b = b + a$
Commutativity of Multiplication	$a \cdot b = b \cdot a$
Associativity of Addition	$a + (b + c) = (a + b) + c$
Associativity of Multiplication	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Additive Identity	$a + 0 = 0 + a = a$
Multiplicative Identity	$a \cdot 1 = 1 \cdot a = a$
Additive Inverse	$a + (-a) = -a + a = 0$
Multiplicative Inverse	$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1, a \neq 0$
Distributivity	$a(b + c) = ab + ac$

Section 1.1 Exercises

1. What is the difference between a rational number and an irrational number?
2. Can a number be *both* rational *and* irrational? If so, give an example. If not, explain why.

Classify the numbers in problems 3–9 as real, rational, irrational, integer, and/or natural.

3. -2
4. $4.\overline{6}$
5. 5
6. $\sqrt{3}$
7. $\sqrt{4}$

8. $\frac{7}{2}$
9. $\frac{\sqrt{7}}{2}$
10. a. Explain why $0.14159265 \dots = \pi - 3$ is an irrational number.
 b. Explain why $0.123456789876543212345678987654321 \dots$ is a rational number. Its decimal representation is the digits 1 through 9 in ascending order, then descending back to 1, and so on.
11. Give the ...
 a. Opposite of 7
 b. Reciprocal of 7
 c. Opposite of $\frac{3}{7}$
 d. Reciprocal of $\frac{3}{7}$
12. Addition of real numbers is commutative, but in general subtraction of real numbers is not. For example, $4 - 7 \neq 7 - 4$. What must be true of a and b so that $a - b = b - a$?
13. Give an example to show why subtraction is not associative. That is, find numbers a , b , and c , and calculate $a - (b - c)$ and $(a - b) - c$.
14. Which of the properties of real numbers allows us to write that $48 + 96 = 24(2 + 4)$?
15. Which of the properties of real numbers allows us to FOIL? *FOIL* stands for “first, outer, inner, last” and it’s how we multiply out $(a + b)(c + d)$. We get $(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd$.
16. Which of the properties of real numbers allows us to add four numbers from left to right in any order we want?
17. Which of the properties of real numbers tells us that if we’re multiplying -37 and $\frac{67}{11}$, then we can write down either number first?
18. Is division commutative for all real numbers (except zero)? Explain why or why not.
19. For a property to be true, it needs to hold for all real numbers.
 a. Suppose that division has an associative property. This means we claim that

$$a \div (b \div c) = (a \div b) \div c.$$

Calculate both sides of this equation when $a = 2$, $b = 1$, and $c = 1$. Does this mean the property holds for all real numbers?

- b. Find different numbers for a , b , and c , calculate both sides of the so-called associative property of addition, and show that this property does not hold for all real numbers.
20. Looking ahead: Write down the sample space that describes the outcomes when a fair 6-sided die is rolled. Meaning, write down a set Ω that lists the possibilities when rolling the die.

1.2: THE FOUR BASIC OPERATIONS

In this section we discuss the four basic operations on real numbers: addition, subtraction, multiplication, and division. We also talk about another operation: absolute value.

For the most part, we will discuss these operations in the context of rational numbers. (We will discuss how these operations act on irrational numbers in detail in Chapter 7.)

In the last part of this section, we will discuss how the four operations act together in mathematical expressions.

Addition and Subtraction

When adding two positive integers, the result is positive.

Ex) $37 + 750 = 787$

One convention that we adopt is that if a and b are real numbers, then

$$a + (-b) = a - b.$$

So adding the opposite of a number is really like subtraction. Therefore, when we add two negative integers, the result is negative.

Ex) $-3 + (-7) = -3 - 7 = -10$

When adding one positive and one negative integer, follow the rules discussed in the previous section, remembering that adding a negative is like subtracting.

Ex) $-3 + 7 = 4$. The result is positive, because 7 is larger than 3.

Ex) $6 + (-15)$ is the same as $6 - 15 = -9$. The result is negative because 15 is larger than 6.

When adding or subtracting negative numbers, we have to remember that subtracting a negative number is like adding a positive number: We are reversing an opposite.

Ex) If we are solving for $-7 - (-12)$, we are taking away the opposite of 12. Think of the opposite of taking away as giving back, so we are really adding. So $-7 - (-12) = -7 + 12 = 5$.

Ex) Suppose you have \$10, and your friend takes away the opposite of \$5. This is like your friend giving you \$5. In mathematical notation, you have $10 - (-5) = 15$ dollars.

Below is a summary of what we have discussed.

When adding ...	the result is ...
two positive numbers	positive
one positive number and one negative number	like computing $a - b$ where $a, b > 0$; the result is positive if $a > b$ and negative if $b > a$
two negative numbers	negative

When adding and subtracting rational numbers, it is important to have a common denominator.

Ex) If one person eats one and one-half pies, and another eats one-sixth of a pie, how much pie is eaten? Well, consider that one and one-half is $1\frac{1}{2}$. Writing this as a complex fraction, we have $1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$. Then, adding, we have $\frac{3}{2} + \frac{1}{6} = \frac{3}{2} \cdot \frac{3}{3} + \frac{1}{6} = \frac{9}{6} + \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$. So in total, five-thirds pies are eaten.

Ex) $4 - \frac{7}{2} = \frac{4}{1} \cdot \frac{2}{2} - \frac{7}{2} = \frac{8}{2} - \frac{7}{2} = \frac{1}{2}$

Multiplication and Division

When multiplying a times b , think of having a copies of b . So if we have $2 \cdot 3$, then we have 2 sets of 3 objects, or a total of 6 objects. This also works well if you are multiplying one positive number and one negative number.

Ex) $-3 \cdot 2 = -6$, because we have 2 copies of -3 , or -6 in total.

Ex) $4(-6)$ is like 4 copies of -6 , so there is a total of -24 .

How does this make sense when we multiply two negative numbers? Why does $(-2)(-3) = 6$? The answer goes back to reversing opposites. If “taking away” is a negative, then the reverse of “taking away” is actually “giving,” or a positive.

Ex) Suppose you pay rent out of your bank account. Think of $2(-700)$ as you making two rent payments of \$700 each, so you are taking away \$1400 from your account. Then what does $(-2)(-700)$ represent? It is the opposite of deducting two rent payments from your account, so you are actually adding \$1400 to your account. If only it were that easy.

When multiplying rational numbers, we take the product of the numerators divided by the product of the denominators.

Ex) If we multiply $\frac{2}{3}$ times $\frac{3}{2}$, we have $\frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1$.

Ex) If we multiply 6 times $\frac{3}{5}$, we have $\frac{6}{1} \cdot \frac{3}{5} = \frac{6 \cdot 3}{1 \cdot 5} = \frac{18}{5}$.

Ex) $-7\left(\frac{3}{14}\right) = -\frac{7}{1} \cdot \frac{3}{14} = -\frac{3}{2}$

The same rules apply to division. If only one of the numbers that we are dividing is negative, the result is negative. However, if both numbers are negative, the result is positive.

Ex) $-4 \div 2 = -2$

Ex) $-4 \div (-2) = 2$, since we are dividing two negative numbers.

Below is a summary of what we've discussed.

When multiplying or dividing ...	the result is ...
two positive numbers	Positive
one positive and one negative number	Negative
two negative numbers	Positive

When dividing fractions, remember that dividing by a number is the same as multiplying by the reciprocal of that number. So the second fraction is flipped.

$$\text{Ex) } \frac{2}{3} \div \frac{3}{2} = \frac{2}{3} \cdot \frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 3} = \frac{4}{9}$$

$$\text{Ex) } \frac{2}{3} \div \left(-\frac{5}{3}\right) = \frac{2}{3} \cdot \left(-\frac{3}{5}\right) = \frac{2(-3)}{3(5)} = -\frac{2}{5}$$

Ex) Dividing $-\frac{1}{3}$ by $-\frac{8}{9}$ gives us

$$\begin{aligned} -\frac{1}{3} \div \left(-\frac{8}{9}\right) &= -\frac{1}{3} \cdot \left(-\frac{9}{8}\right) \\ &= \frac{1 \cdot 9}{3 \cdot 8} \\ &= \frac{3}{8}. \end{aligned}$$

A helpful tip: When you're multiplying fractions like this, simplify as you go. For example, in the second-to-last step above, you want to divide the 3 into the 9 to get the answer of $\frac{3}{8}$, rather than multiplying straight across to get $\frac{9}{24}$ and having to simplify now.

At this point, it is important to note what negatives or opposites do to fractions. If you are dividing 4 by -1 , the result is -4 . But there are a few ways to write “ -1 divided by 4.” The first way is to have the negative sign sitting in the numerator, as in $\frac{-1}{4}$. The second way is to move the negative out front, as in $-\frac{1}{4}$ (this is the way this text will write negative fractions most often). The third way is to have the negative sign move to the denominator, as in $\frac{1}{-4}$ (this way is the least common, but all three are equally correct). Again, we have

$$\frac{-1}{4} = -\frac{1}{4} = \frac{1}{-4}.$$

We'll refer to this again when we talk about rational expressions in Section 6.2.

Absolute Value

On a number line, the starting point zero is sometimes called the origin. The absolute value of a number is the distance from the number to the origin. The notation for absolute value is to put vertical bars around the number. So the absolute value of a number x is written as $|x|$.

Ex) Since the number 5 is five units away from zero, we have $|5| = 5$. See Figure 1-4.

Ex) Since the number -5 is five units away from zero, we have $|-5| = 5$. See Figure 1-4.

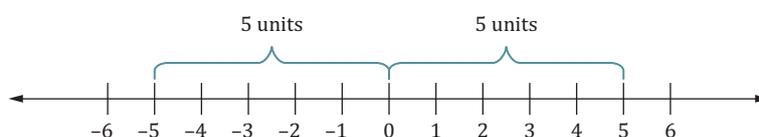


Fig. 1-4
 $|5| = 5$ and $|-5| = 5$

We can summarize this by saying that if x is positive, then $|x| = x$, but if x is negative, then $|x| = -x$.

Ex) In $|-16|$, we are taking the absolute value of a negative number. So the answer is the opposite of this number, or $-(-16) = 16$.

Ex) $\left| -\frac{4}{7} \right| = \frac{4}{7}$

It is helpful to use absolute value when subtracting numbers. When you're subtracting numbers and trying to determine if the result is positive or negative, you keep the sign of the number that has the highest absolute value.

Ex) In the equation $4 - \frac{9}{2}$, since $\frac{9}{2}$ has a higher absolute value than 4, we know the answer will be negative. The result is $4 - \frac{9}{2} = \frac{8}{2} - \frac{9}{2} = -\frac{1}{2}$.

The Four Operations Together

If we look at a mathematical expression written out, like $1 + 1 \div 4$, it might seem that we are to add 1 and 1 together first. But if we are asked to add one and one-fourth, then we could write this out mathematically as $1 + \frac{1}{4}$. It is important for us to be consistent with notation, so we need to make sure that $1 + 1 \div 4$ and $1 + \frac{1}{4}$ have the same numerical value: $\frac{5}{4}$. This is why

multiplication and division are done from left to right before addition/subtraction.

So when we're simplifying mathematical expressions, multiplication and division are done from left to right in the order they appear. And after that, addition and subtraction operations are performed from left to right in the order they appear.

To the examples!

Example: Simplify $2 \div 1 \cdot 0$.

Solution: Solving from left to right, we have $2 \div 1 \cdot 0 = 2 \cdot 0 = 0$.

Example: Simplify $1 - 1 + 1 - 2 \cdot 0$.

Solution: A good first step when simplifying is to scan the expression. Ignore the addition and subtraction for a moment, because the operation that is done first is the multiplication. It also helps to place parentheses around the two numbers that are affected by the operation, to make it clear what needs to happen. So here is how we should do this simplification:

$$\begin{aligned} 1 - 1 + 1 - 2 \cdot 0 &= 1 - 1 + 1 - (2 \cdot 0) \\ &= 1 - 1 + 1 - 0 \\ &= (1 - 1) + 1 - 0 \\ &= 0 + 1 - 0 \\ &= 1. \end{aligned}$$

Example: Simplify $2 \div 4 - 4 \div 2$.

Solution: A good way to think of this is $\frac{2}{4} - \frac{4}{2}$, which equals $\frac{1}{2} - 2$. Then we can consider the solution as just a matter of subtracting fractions, in which case we have $\frac{1}{2} - 2 = \frac{1}{2} - \frac{4}{2} = -\frac{3}{2}$.

Example: Simplify $14 + 20 \div 2 \div 2 + 6 \cdot 2 - 3$.

Solution: The first thing to take care of is the division. Then we move on to the other division and the multiplication. Again, it is a good idea to use parentheses to highlight what operation is happening next. This is how we should simplify:

$$\begin{aligned}
 14 + 20 \div 2 \div 2 + 6 \cdot 2 - 3 &= 14 + (20 \div 2) \div 2 + 6 \cdot 2 - 3 \\
 &= 14 + 10 \div 2 + 6 \cdot 2 - 3 \\
 &= 14 + (10 \div 2) + 6 \cdot 2 - 3 \\
 &= 14 + 5 + (6 \cdot 2) - 3 \\
 &= 14 + 5 + 12 - 3 \\
 &= 19 + 12 - 3 \\
 &= 31 - 3 \\
 &= 28.
 \end{aligned}$$

As you do more and more of these problems, you will need fewer and fewer steps. For example, you will not need to note the parentheses before you do every operation. But for now, adding parentheses is a great way to highlight the order of steps taken.

Example: Simplify $13 + 5 - (-2) - 5 - (-6) + 10$.

Solution: Since this expression contains only addition and subtraction, we move from left to right in successive steps:

$$\begin{aligned}
 13 + 5 - (-2) - 5 - (-6) + 10 &= 18 - (-2) - 5 - (-6) + 10 \\
 &= 18 + 2 - 5 - (-6) + 10 \\
 &= 20 - 5 - (-6) + 10 \\
 &= 15 - (-6) + 10 \\
 &= 15 + 6 + 10 \\
 &= 21 + 10 \\
 &= 31.
 \end{aligned}$$

Example: Simplify $64 \div 4 \cdot (-2) \div 8 \cdot (-3)$.

Solution: Again, since this expression contains only multiplication and division, we move from left to right in successive steps:

$$\begin{aligned} 64 \div 4 \cdot (-2) \div 8 \cdot (-3) &= 16 \cdot (-2) \div 8 \cdot (-3) \\ &= -32 \div 8 \cdot (-3) \\ &= -4 \cdot (-3) \\ &= 12. \end{aligned}$$

Example: Simplify $|-3| - |5 - 7| + \left| \frac{1}{2} - 3 \right|$.

Solution: Simplifying inside the absolute values first, we find this:

$$\begin{aligned} |-3| - |5 - 7| + \left| \frac{1}{2} - 3 \right| &= 3 - |-2| + \left| \frac{1}{2} - \frac{6}{2} \right| \\ &= 3 - 2 + \left| -\frac{5}{2} \right| \\ &= 1 + \frac{5}{2} \\ &= \frac{2}{2} + \frac{5}{2} \\ &= \frac{7}{2}. \end{aligned}$$

A last comment: it is worth noting that there are several ways to signify multiplication: using the dot, \cdot ; using the “times” symbol, \times ; using the asterisk, $*$; and using parentheses (parentheses are popular when we are dealing with negative numbers). To indicate division, we use a fraction bar or the division symbol, \div .

Example: Simplify $1 \cdot 2 \times 3 * 4 \div 6 \cdot \frac{1}{4}$.

Solution: From left to right, we have $2 \times 3 * 4 \div 6 \cdot \frac{1}{4} = 6 * 4 \div 6 \cdot \frac{1}{4} = 24 \div 6 \cdot \frac{1}{4} = 4 \cdot \frac{1}{4} = 1$.

Section 1.2 Exercises

In problems 1–38, simplify the expression.

- $1 + \frac{3}{7}$
- $1 + (-2)$
- $\frac{3}{8} + \frac{2}{5}$
- $4 - 8$
- $47 - 56$
- $-4 - (-3)$
- $\frac{2}{3} + \left(-\frac{2}{3}\right)$
- $\frac{2}{3} - \left(-\frac{2}{3}\right)$
- $4 \cdot \frac{7}{2}$
- $-2 \cdot 5$
- $-2 \cdot (-5)$
- $-2 \div 1$
- $-2 \div (-1)$
- $\frac{3}{23} \cdot \frac{46}{9}$
- $\frac{3}{7} \cdot \frac{14}{27}$
- $\frac{14}{15} \cdot \left(-\frac{5}{2}\right)$
- $-\frac{14}{15} \cdot \left(-\frac{5}{2}\right)$
- $\frac{14}{15} \div \frac{5}{2}$
- $\frac{14}{15} \div \left(-\frac{5}{2}\right)$
- $-\frac{14}{15} \div \left(-\frac{5}{2}\right)$
- $|-3|$
- $|3|$
- $\left|\frac{3}{8}\right|$
- $|0|$
- $\left|-\frac{2}{9}\right|$

26. $|-5| + 7 \cdot 2 - 6$
27. $\left|-\frac{1}{3}\right| + \frac{5}{6}$
28. $4 \cdot 5 \div 2$
29. $3 - 6 \cdot 5$
30. $18 \div 2 \div 3$
31. $2 \cdot 3 \cdot 4 \cdot 5 \cdot 0$
32. $4 + 7 - 3 \cdot 10 \div 2$
33. $3 \cdot 4 - 8 \cdot 2$
34. $18 \div 2 - 15 \div (-3)$
35. $3 \div 7 + 4 \div 7$
36. $14 - 16 \div 4 + 2 \cdot 6 \div 3$
37. $64 \div 2 \div 2 \div 2$
38. $2 \div 2 \div 2 \div 2$

1.3: ORDER OF OPERATIONS—PEMDAS

In this section we will discuss natural number exponents. After that, we will be prepared to handle the classical order of operations.

Exponents—A First Look

Exponents are a shorthand for multiplication. They are very helpful if we need to multiply a number by itself several times. We have the following definition:

$$x^n = x \cdot x \cdot \dots \cdot x \text{ (} n \text{ times)}.$$

In words, we say that “ x to the n th power is x multiplied by itself n times.”

Ex) 3^4 is 4 copies of 3 multiplied out, so we have $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 9 \cdot 3 \cdot 3 = 27 \cdot 3 = 81$.

Taking the second power and third power of a number has a direct reference to geometry. When we are raising a number to the second power, we are squaring it, so we write $6^2 = 36$, and we say

“6 squared equals 36.” When we are raising a number to the third power, we are cubing it, so we write $2^3 = 8$ and we say “2 cubed equals 8.”

$$\text{Ex) } (-1)^5 = (-1)(-1)(-1)(-1)(-1) = 1(-1)(-1)(-1) = 1(-1) = -1$$

When we raise fractions to powers, we put parentheses around them.

$$\text{Ex) } \left(\frac{2}{5}\right)^3 = \frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5} = \frac{2 \cdot 2 \cdot 2}{5 \cdot 5 \cdot 5} = \frac{8}{25}$$

$$\text{Ex) } \left(\frac{3}{4}\right)^2 = \frac{3}{4} \cdot \frac{3}{4} = \frac{3 \cdot 3}{4 \cdot 4} = \frac{9}{16}$$

When we raise a fraction to a power, we raise the numerator to that power and the denominator to that power:

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0.$$

$$\text{Ex) } \left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$$

Example: Simplify $\left(-\frac{1}{5}\right)^2$.

Solution: Recall our discussion about fractions and where we can place the negative sign. When squaring, move the negative to the top or the bottom number. The negative is definitely affected by the exponent:

$$\begin{aligned} \left(-\frac{1}{5}\right)^2 &= \left(\frac{-1}{5}\right)^2 \\ &= \frac{(-1)^2}{5^2} \\ &= \frac{1}{25}. \end{aligned}$$

When the fraction has a low exponent like this one does, we could have just written out

$$\left(-\frac{1}{25}\right)\left(-\frac{1}{25}\right) = \frac{1}{25}.$$

Example: Simplify $\left(-\frac{2}{3}\right)^5$.

Solution:

$$\begin{aligned}\left(-\frac{2}{3}\right)^5 &= \frac{(-2)^5}{3^5} \\ &= -\frac{32}{243}.\end{aligned}$$

YOU TRY IT: Evaluate $\left(-\frac{2}{5}\right)^3$.

PEMDAS

Let P stand for “parentheses,” E stand for “exponents,” M for “multiplication,” D for “division,” A for “addition,” and S for “subtraction.” We use the acronym PEMDAS, which stands for “Please Excuse My Dear Aunt Sally,” to remember the order in which general expressions are simplified. We already covered the MDAS portion of PEMDAS in the last section. Multiplication and division are done from left to right, and then addition and subtraction are done last from left to right. But before doing any of these operations, we must take care of parentheses and exponents. Parentheses are to be worked from the inside out, and then exponents are resolved. Then we move on from there.

Example: Simplify $3 + 2^2 \cdot 4$.

Solution: We must square 2 before doing the multiplication, so we have

$$\begin{aligned}3 + 2^2 \cdot 4 &= 3 + 4 \cdot 4 \\ &= 3 + 16 \\ &= 19.\end{aligned}$$

Example: Simplify $(-2)^2 - 2^2$.

Solution: Taking care of the parentheses first, we square -2 to get $4 - 2^2$. Then, we must take the exponent before we subtract, so we have $4 - 2^2 = 4 - 4 = 0$. A common mistake is made in evaluating -2^2 ; note that this is the opposite of 2^2 , so it is -4 .

Example: Simplify $-2(3 - (2 - 5))$.

Solution: When there are multiple groups of parentheses, we work from the inside out, as follows:

$$\begin{aligned} -2(3 - (2 - 5)) &= -2(3 - (-3)) \\ &= -2(3 + 3) \\ &= -2(6) \\ &= -12. \end{aligned}$$

YOU TRY IT: Simplify $(-2)^2 + 2^2(-3)$.

Having a lot of parentheses can be confusing to the eye. So brackets and braces are used as well: [] and { }, respectively. These have the same priority of parentheses, and we work from the inside out.

Ex) $-2(-3(2 + 7(-3 + 2)))$ can be written as $-2\{-3[2 + 7 + (-3 + 2)]\}$, which lets us clearly see what is “inside” (and therefore where to start simplifying).

Example: Simplify $-2\{-3[2 + 7 + (-3 + 2)]\}$.

Solution: Start with the braces and work your way out:

$$\begin{aligned} -2\{-3[2 + 7 + (-3 + 2)]\} &= -2\{-3[2 + 7(-1)]\} \\ &= -2\{-3[2 - 7]\} \\ &= -2\{-3[-5]\} \\ &= -2\{15\} \\ &= -30. \end{aligned}$$

Example: Simplify $4(-2)^2$.

Solution: A good habit is to say what this is by using words. It is “4 times negative 2 squared.” The parentheses make clear that we must square -2 first, so it is even better to think of this as “4 times the square of -2 .” Thus, we have $4(-2)^2 = 4(4) = 16$.

When simplifying fractions, we can think of $\frac{1+3-2}{4+6\cdot 2}$, for example, as $(1+3-2) \div (4+6\cdot 2)$. Because of the parentheses, we can simplify the numerator and the denominator separately.

$$\text{Ex) } \frac{1+3-2}{4+6\cdot 2} = \frac{4-2}{4+12} = \frac{2}{16} = \frac{1}{8}$$

Section 1.3 Exercises

Simplify.

1. $1+8-4\cdot 5$
2. $-2\div |-5|\cdot 10$
3. $1+2\cdot 3-4\div 2$
4. $4-6^2(5-17)$
5. $18\div (3^2-6)$
6. $\frac{4}{3}+2|-4-7|$
7. $0.6+3^2(1-(1-2))$
8. $4-7[3-2(5-6)+2^2]$
9. $2-18\left\{\left(\frac{1}{2}\right)^2\cdot 4^2+1\right\}$
10. $\left(-\frac{3}{5}\right)^2+\left(\frac{1}{5}\right)^3$
11. $\left(\frac{2}{3}-1^2\right)\left(4^2-\frac{2}{3}\right)$
12. $\frac{4-5\cdot 2}{18\div 9+7\cdot 3-2}$
13. $\frac{5-6(3-2^2)}{14-12\div 3}$

1.4: ORDER OF OPERATIONS—LIKE TERMS AND GROUPING SYMBOLS

In this section we will use like terms to simplify mathematical expressions. After that, we will look at how grouping symbols interact with these expressions by use of a major property from Section 1.1.

Like Terms

We begin with notation that will make the discussion of these and further topics a lot easier. An expression is a mathematical grouping of numbers and symbols with no equal sign. For example, the absolute value of x , $|x|$, is an expression.

Ex) $x^2 + 5x + 6$ is an expression.

Ex) $1 + 2 + 3$ is an expression.

Now, in an expression there are terms. Terms are separated by addition. For example, in the expression $x^2 + 5x + 6$, the three terms are x^2 , $5x$, and 6 .

Even if we are subtracting, we separate the terms with addition.

Ex) Consider $1 + 2x - x^3$. Using the rule that subtraction is adding opposites, we think of this as $1 + 2x + (-x^3)$. So the terms of $1 + 2x - x^3$ are 1 , $2x$, and $-x^3$.

YOU TRY IT: Write down the terms of $x^4 - x^3 + 1$.

We will be spending a lot of time in this text simplifying expressions. So it is crucial to know *when* expressions can be simplified. It stands to reason that if you have $x^2 + x^2$, you are adding one x^2 to another x^2 , so you have two of them. Indeed, like terms have the same variable (or variables) raised to the same power. If a constant is multiplied by these variables, we call the resulting number the coefficient of the term.

Ex) In the expression $x^2 + x^3 - 2x^2 + 4x^3$, one set of like terms is x^2 and $-2x^2$. Also note that x^3 and $4x^3$ are like terms. The coefficient of x^2 is 1 , the coefficient of $-2x^2$ is -2 , and the coefficient of $4x^3$ is 4 .

YOU TRY IT: Write down the terms of $2x^2 + 3xy - y^2 + 6y$ and their coefficients.

When we are adding like terms, we add the coefficients of the terms. This way, if we are adding $x^2 + x^2$, we are adding one x^2 to another x^2 , which makes $2x^2$. This is all thanks to the distributive property, which we covered in Section 1.1. Remember that the distributive property tells us that $a(b + c) = ab + ac$. Well we are using this property from the right side to the left side. Think of it as

$$ba + ca = (b + c)a.$$

Then, using words, think of this as “ b copies of a plus c copies of a .” Then we have, in total, “ $b + c$ copies of a .”

Example: Simplify $x^2 + x^3 - 2x^2 + 4x^3$.

Solution: We have

$$\begin{aligned} x^2 + x^3 - 2x^2 + 4x^3 &= x^2 - 2x^2 + x^3 + 4x^3 \\ &= (1 - 2)x^2 + (1 + 4)x^3 \\ &= (-1)x^2 + (5)x^3 \\ &= -x^2 + 5x^3. \end{aligned}$$

Here we are using the convention that -1 times a variable is the opposite of that variable. That is, $(-1)x^2$ is the same thing as $-x^2$.

Ex) To simplify $4y + x - 7y$, note that the y 's are like terms, but the x must be left alone. So we can add the coefficients 4 and -7 to get -3 . Thus, $4y + x - 7y = -3y + x$. Since addition is commutative, the answer $-3y + x$ is the same as the answer $x - 3y$.

When we begin to simplify complicated expressions, keep track by making marks under the like terms as you simplify them.

Example: Simplify $2y - 2x + 6x^2 - 3y + 3x - 8x^2$.

Solution: To keep things clear, let's underline the like terms that we are adding in each step. We have

$$\begin{aligned} \underline{2y} - 2x + 6x^2 - \underline{3y} + 3x - 3x^2 &= -y - \underline{2x} + 6x^2 + \underline{3x} - 8x^2 \\ &= -y + x + \underline{6x^2} - \underline{8x^2} \\ &= -y + x - 2x^2. \end{aligned}$$

Example: Simplify $3x^2 + 6xy - 5y^2 - 15xy + 10y^2 - 8x^2$ by combining like terms.

Solution: Underlining like terms as we go, we have

$$\begin{aligned} \underline{3x^2} + 6xy - 5y^2 - 15xy + 10y^2 - \underline{8x^2} &= -5x^2 + \underline{6xy} - 5y^2 - \underline{15xy} + 10y^2 \\ &= -5x^2 - 9xy - \underline{5y^2} + \underline{10y^2} \\ &= -5x^2 - 9xy + 5y^2. \end{aligned}$$

Example: Simplify $3x^2 + 7x^2y - 15y^2 + 13xy^2$ by combining like terms.

Solution: Take a second to look at the variables. We have an x^2 term, an x^2y term, a y^2 term, and an xy^2 term (with various coefficients). But there are no like terms. This expression is as simplified as it can be.

Example: Simplify $3x^2 + 7x^2y + 6xy^2 - 8xy^2 - 15x^2 + 3xy^2$.

Solution: Let's underline the like terms as we simplify. We have

$$\begin{aligned} 3x^2 + 7x^2y + 6xy^2 - 8xy^2 - 15x^2 + 3xy^2 &= \underline{3x^2} + 7x^2y + 6xy^2 - 8xy^2 - \underline{15x^2} + 3xy^2 \\ &= -12x^2 + 7x^2y + \underline{6xy^2} - \underline{8xy^2} + \underline{3xy^2} \\ &= -12x^2 + 7x^2y + xy^2. \end{aligned}$$

Example: Simplify $\frac{1}{2}x^3 - 0.3x^2 + \frac{2}{5}x^3 + 1.6x^2$.

Solution: We add the coefficients of x^3 together. So we make a side note that

$$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{5+4}{10} = \frac{9}{10}.$$

So we have

$$\begin{aligned} \frac{1}{2}x^3 - 0.3x^2 + \frac{2}{5}x^3 + 1.6x^2 &= \frac{1}{2}x^3 - 0.3x^2 + \frac{2}{5}x^3 + 1.6x^2 \\ &= \frac{9}{10}x^3 - \underline{0.3x^2} + \underline{1.6x^2} \\ &= \frac{9}{10}x^3 + 1.3x^2. \end{aligned}$$

Grouping Symbols

We can extend the distributive property to more than two numbers. For example, if we need to multiply $2(3 + 5 + 8)$, we can add $2 \cdot 3 + 2 \cdot 5 + 2 \cdot 8$. In fact, the distributive property will work for any number of values:

$$a(b + c + d + e + \cdots + z) = ab + ac + ad + ae + \cdots + az.$$

For emphasis, we usually draw arrows from the number we are distributing to each term in the parentheses.

Example: Simplify $-4(3 + 5 + 7)$.



$$-4(3 + 5 + 7)$$

Fig. 1-5

Solution: This is $-4 \cdot 3 + (-4) \cdot 5 + (-4) \cdot 7 = -12 - 20 - 28 = -32 - 28 = -60$. See Figure 1-5.

Example: Simplify $-(8 - 2)$.

Solution: If we distribute a negative sign, we imagine distributing a -1 . So we have

$$\begin{aligned} -(8 - 2) &= -1(8 - 2) \\ &= -1 \cdot 8 - 1 \cdot (-2) \\ &= -8 - 1(-2) \\ &= -8 + 2 \\ &= -6. \end{aligned}$$

This is an interesting take on simplifying. Note that when we discussed order of operations in the previous section, we demanded simplifying inside the parentheses first. This is one of the nice things about mathematics: There is often more than one way to find the solution to a problem.

Of course, we need to be able to distribute when we have variable expressions. For example, if we wanted to double $x + y$, the expression would be $2(x + y)$. By the distributive property, this is $2x + 2y$. So note that when you distribute a number to an expression, you are multiplying that number by the *coefficients* of the terms of that expression.

Example: Simplify $-3(x + 2y)$.

Solution: Distributing -3 , we have $-3x + (-3)(2y) = -3x - 6y$.

If the term has a coefficient of -1 , we again use the convention that there is an imaginary -1 in front of the term.

Example: Simplify $2(b - ab - 15b^2)$.

Solution: Distributing the 2 gives $2b + 2(-1ab) + 2(-15b^2) = 2b - 2ab - 30b^2$.

Example: Simplify $\frac{1}{2}\left(4x^2 - \frac{1}{2}x + \frac{4}{9}\right)$.

Solution: We have

$$\begin{aligned}\frac{1}{2}\left(4x^2 - \frac{1}{2}x + \frac{4}{9}\right) &= \frac{1}{2}(4x^2) + \frac{1}{2}\left(-\frac{1}{2}x\right) + \frac{1}{2}\cdot\left(\frac{4}{9}\right) \\ &= \frac{1}{2}\cdot\frac{4}{1}x^2 - \frac{1}{2}\cdot\frac{1}{2}x + \frac{1}{2}\cdot\frac{4}{9} \\ &= 2x^2 - \frac{1}{4}x + \frac{2}{9}.\end{aligned}$$

Example: Simplify $0.2(4x - 3y^2)$.

Solution: Distributing the 0.2, we have $0.2(4x) + 0.2(-3y^2) = 0.8x - 0.6y^2$.

YOU TRY IT: Simplify $14\left(2x - 3y + \frac{1}{7}z\right)$.

Example: Simplify $-4(3 - 2(x - y))$.

Solution: We are combining all our knowledge of the order of operations here. Start from the inside, and distribute -2 :

$$\begin{aligned}-4(3 - 2(x - y)) &= -4(3 - 2 \cdot x - 2(-y)) \\ &= -4(3 - 2x + 2y) \\ &= -4 \cdot 3 - 4(-2x) - 4(2y) \\ &= -12 + 8x - 8y.\end{aligned}$$

YOU TRY **Solution:** Terms are $2x^2$, $3xy$, $-y^2$, and $6y$; coefficients are 2, 3, -1 , 6.

Section 1.4 Exercises

- What is an expression?
 - What is a term?
 - What is a coefficient?
- How many terms are in the expression $3x^3 + 4x^2 - 5x + 6$?
 - List the terms and their coefficients.
- How many terms are in the expression $0.3x - 5y + 2z$?
 - List the terms and their coefficients.

4. Simplify $2x + 3y - 5x - 10y$.
5. Simplify $\frac{1}{2}x + \frac{1}{3}y - \frac{1}{5}x - \frac{1}{10}y$.
6. Simplify $0.3x + 1.2y - 2x - 0.5y$.
7. Simplify $2x - x^2 + 3x + 2x^2 - 5$.
8. Simplify $x^2 + 2x + 1 + y^2 + 2y + 7 - 3x^2 - 2y$.
9. Simplify $\frac{1}{2}x^2 + 4y^2 - \frac{3}{2}x^2 - y^2$.
10. Simplify $4x^2y - 3xy^2 + 15y^2 + 10xy^2 - 8y^2 + 2x^2y$.
11. Simplify $4ab + 2b - 8a + 10ab - 18a + 16b$.
12. Simplify $\frac{-49x - 3y(7 - 4)}{y + 3(2 - y)}$.
13. Simplify $\frac{1}{8}(5 - 3)$.
14. Simplify $7(4x - 3y)$.
15. Simplify $-2\left(x + \frac{1}{2}y\right)$.
16. Simplify $-2(3 - 5x(a + 2))$.
17. Simplify $x(5 - 3^2)$.
18. Simplify $2 - [1 - 3(y - x)]$.
19. Simplify $\frac{1}{2}\left[4 - \frac{1}{3}(x - 2)\right]$.

1.5: RULES FOR EXPONENTS

In this section we will discuss the rules for exponents, which are used in every math class—and are the cause of many common errors. We will discuss expressions that have exponents that are integers.

Integer Exponents

For the expression a^m , we call the number a the base, and we call m the exponent. As an example, in the expression 2^3 , the base is 2 and the exponent is 3.

In Section 1.3, we talked about natural number exponents. So we know that if n is positive, then

$$x^n = x \cdot x \cdot \cdots \cdot x \text{ (} n \text{ times)}.$$

But what happens if we raise a number to the “zero power”? We have to choose a convention here, and because raising a number to a positive power deals with multiplication, it makes sense that when we raise a number to the zero power we get the multiplicative identity.

Any nonzero number raised to the zero power is 1.

Ex) We write $2^0 = 1$, and $(-2)^0 = 1$. Note carefully, however, that -2^0 is the opposite of 2^0 , which is -1 .

Ex) Similarly with variables, we have $x^0 = 1$, as long as $x \neq 0$, and we have $(a + b)^0 = 1$, as long as $a + b \neq 0$.

Now, in keeping with the ideas of multiplication and the multiplicative identity, we will define what negative exponents are:

Any nonzero number raised to the -1 power is its reciprocal.

Ex) We have $2^{-1} = \frac{1}{2}$.

Ex) Note that if we raised 0 to the -1 power we would have $0^{-1} = \frac{1}{0}$, and division by zero is not allowed. Both 0^{-1} and $\frac{1}{0}$ are undefined.

Example: Simplify $3(4 + 2^{-1})$.

Solution: Simplify the inside first, to get $3(4 + 2^{-1}) = 3\left(4 + \frac{1}{2}\right) = 3\left(\frac{8}{2} + \frac{1}{2}\right) = 3\left(\frac{9}{2}\right) = \frac{3}{1} \cdot \frac{9}{2} = \frac{27}{2}$.

Note that we could also distribute the 3 like we did in Section 1.4.2.

If we understand a negative exponent to mean “reciprocal,” we need to take one more step to make our definition of exponents for all integers. That is,

$$x^{-n} = \frac{1}{x^n}, \quad x \neq 0, \quad n \in \mathbb{Z}.$$

Example: Simplify $2^{-2} - 3^{-2}$.

Solution: $\frac{1}{2^2} - \frac{1}{3^2} = \frac{1}{4} - \frac{1}{9} = \frac{1}{4} \cdot \frac{9}{9} - \frac{1}{9} \cdot \frac{4}{4} = \frac{9-4}{36} = \frac{5}{36}$

What happens when a fraction is raised to a negative number? The rule states that a number raised to the -1 power is its reciprocal, and the reciprocal of a fraction switches the numerator and the denominator. Thus, for any fraction $\frac{a}{b}$, we have

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}.$$

Next, if we take a fraction to an integer less than -1 , we first take care of the reciprocal and then simplify as normal.

Example: Simplify $\left(\frac{2}{3}\right)^{-2} - 3^{-2}$.

Solution: We need to make the exponents positive so we can start simplifying, so we take reciprocals first:

$$\begin{aligned} \left(\frac{2}{3}\right)^{-2} - 3^{-2} &= \left(\frac{3}{2}\right)^2 - \frac{1}{3^2} \\ &= \frac{3^2}{2^2} - \frac{1}{9} \\ &= \frac{9}{4} - \frac{1}{9} \\ &= \frac{9}{4} \cdot \frac{9}{9} - \frac{1}{9} \cdot \frac{4}{4} \\ &= \frac{81}{36} - \frac{4}{36} \\ &= \frac{77}{36}. \end{aligned}$$

YOU TRY IT: Simplify $\left(\frac{2}{5}\right)^{-2} + \left(\frac{2}{3}\right)^{-1}$.

Exponent Rules

Working with the above definitions, we can handle the rest of the rules for exponents.

When multiplying two expressions that have the same base, we add the exponents:

$$a^m a^n = a^{m+n}.$$

If we multiply m copies of a , and then we multiply that number by n copies of a , it stands to reason that there are a total of $m + n$ copies of a being multiplied together.

Ex) $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$. It always helps to imagine it when you can. We have two copies of 2 multiplied by 3 copies of 2, so there are 5 copies:

$$2^2 \cdot 2^3 = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^5 = 32.$$

Ex) $x^4 x^6 = x^{4+6} = x^{10}$

Ex) $y^2 y^{-7} = y^{2+(-7)} = y^{-5} = \frac{1}{y^5}$

Example: Simplify $x \cdot x^3 \cdot x^5$.

Solution: If no exponent is written, we assume an exponent of 1. So we have

$$x^1 x^3 x^5 = x^{1+3+5} = x^9.$$

Ex) If $x \neq 0$, let's simplify $x \cdot x^{-1}$. Since we have the same base, we add the exponents to get $x^{1+(-1)} = x^0 = 1$. This concurs with our definition of raising a number to the -1 power, because we have $x \cdot x^{-1} = x \cdot \frac{1}{x} = \frac{x}{x} = 1$.

When dividing two expressions that have the same base, we subtract the exponents:

$$\frac{a^m}{a^n} = a^{m-n}.$$

Ex) $\frac{2^2}{2^3} = 2^{2-3} = 2^{-1} = \frac{1}{2}$. This is the same thing as canceling when you have

$$\frac{2^2}{2^3} = \frac{2 \cdot 2}{2 \cdot 2 \cdot 2} = \frac{1}{2}.$$

Example: Simplify $\frac{x^{64}}{x^{57}}$.

Solution: Since we are dividing two numbers with the same base, we subtract exponents, so

$$\frac{x^{64}}{x^{57}} = x^{64-57} = x^7.$$

Ex) Simplifying $\frac{x^2}{x^{-2}}$, we have to be careful of the negative exponent in the denominator. So using our division rule results in $\frac{x^2}{x^{-2}} = x^{2-(-2)} = x^{2+2} = x^4$.

We follow the convention to *always leave our answers with positive exponents*. It is usually helpful (that is, easier) to think of positive exponents rather than negative ones.

Working with a negative exponent means we need to start by taking the reciprocal. Taking care of the negative exponent corresponds to moving something from numerator to denominator, or from denominator to numerator. For example, when writing $\frac{1}{x^{-1}}$, we can remove the negative exponent if we move x from the denominator to the numerator. So $\frac{1}{x^{-1}} = x^1 = x$.

Ex) We can simplify $\frac{x^{-6}}{y^{-5}}$ by moving x^6 downstairs and y^5 upstairs. So $\frac{x^{-6}}{y^{-5}} = \frac{y^5}{x^6}$.

Example: Simplify $x(x^2 + xy^2)$.

Solution: We cannot simplify inside of the parentheses, so we must distribute the x , as follows:

$$\begin{aligned} x(x^2 + xy^2) &= x(x^2) + x(xy^2) \\ &= x \cdot x^2 + x \cdot x \cdot y^2 \\ &= x^{1+2} + x^{1+1}y^2 \\ &= x^3 + x^2y^2. \end{aligned}$$

Example: Simplify

$$\frac{x^{-5}}{x^{-7}} \cdot \frac{x^3}{x^4}$$

Solution: There are a number of ways to solve this. But let's multiply fractions, and use our exponent rules. We have

$$\begin{aligned} \frac{x^{-5}}{x^{-7}} \cdot \frac{x^3}{x^4} &= \frac{x^{-5} \cdot x^3}{x^{-7} \cdot x^4} \\ &= \frac{x^{-5+3}}{x^{-7+4}} \\ &= \frac{x^{-2}}{x^{-3}} \\ &= x^{-2-(-3)} \\ &= x^{-2+3} \\ &= x. \end{aligned}$$

YOU TRY IT: Try simplifying the previous example by taking care of the negative exponents first. That is, move things “upstairs” and “downstairs,” and then multiply the fractions.

Example: Simplify $(2x^2)(-3xy^3)(4x^2y)$.

Solution: Note that these terms in parentheses are not “like” terms, so we cannot add them together. But we certainly can multiply them together. We move the coefficients, x 's, and y 's around so they are grouped together:

$$\begin{aligned} (2x^2)(-3xy^3)(4x^2y) &= 2 \cdot (-3) \cdot 4 \cdot x^2 \cdot x \cdot x^2 \cdot y^3 \cdot y \\ &= -24x^{2+1+2}y^{3+1} \\ &= -24x^5y^4. \end{aligned}$$

Example: Simplify $-2x^{-1}(4x^3 + 3x^2 - 10x - 5)$.

Solution: When distributing here, be very careful. Be liberal with parentheses! A negative coefficient and a negative exponent mean different things (they are opposite and reciprocal, respectively). We have

$$\begin{aligned} -2x^{-1}(4x^3 + 3x^2 - 10x - 5) &= -2x^{-1}(4x^3) - 2x^{-1}(3x^2) - 2x^{-1}(-10x) - 2x^{-1}(-5) \\ &= -2 \cdot 4x^{-1+3} - 2 \cdot 3x^{-1+2} - 2(-10)x^{-1+1} - 2(-5)x^{-1} \\ &= -8x^2 - 6x^1 + 20x^0 + 10x^{-1} \\ &= -8x^2 - 6x + 20 + \frac{10}{x}. \end{aligned}$$

In the transition to the last step here, we are being careful with notation: We are not leaving an exponent of 1, something to the zero power is 1, and we make sure to have a positive exponent.

The last rule answers this question: What happens when we raise something to a power that already has an exponent? For example, what is $(x^2)^5$? Well, we are raising x^2 to the fifth power, so this means that we are multiplying 5 copies of x^2 together:

$$x^2 \cdot x^2 \cdot x^2 \cdot x^2 \cdot x^2 = x^{2+2+2+2+2} = x^{10}.$$

The rule is to multiply those exponents of 2 and 5. Thus,

$$(a^m)^n = a^{mn}, a \in \mathbb{R}, m, n \in \mathbb{Z}.$$

Ex) $(2^2)^3$ is the cube of 2^2 , so we can write this as $2^{2 \cdot 3} = 2^6 = 64$.

Ex) The same rule applies to a product being raised to a power. If we have $(x^2y^2)^3$ this is 3 copies of x^2y . So we multiply 3 by all of the exponents: $(x^2y)^3 = (x^2)^3(y)^3 = x^{2 \cdot 3}y^{1 \cdot 3} = x^6y^3$.

Example: Simplify $x^3(2x^{-2}y^5)^4$.

Solution: Remembering the order of operations, we must apply the exponent of 4 first:

$$\begin{aligned} x^3(2x^{-2}y^5)^4 &= x^3 \cdot 2^4(x^{-2})^4(y^5)^4 \\ &= x^3 \cdot 2^4 \cdot x^{-2 \cdot 4} \cdot y^{5 \cdot 4} \\ &= x^3 \cdot 16 \cdot x^{-8} \cdot y^{20} \\ &= 16 \cdot x^3 \cdot x^{-8} \cdot y^{20} \\ &= 16x^{-5}y^{20}. \end{aligned}$$

Note that the multiplication symbol in the last step is suppressed, because multiplication is implied.

The following is a summary of the rules in this section.

In words	In mathematical symbols
Anything to the zero power is 1 (except 0).	$a^0 = 1, a \neq 0$
A negative exponent means reciprocal.	$a^{-n} = \frac{1}{a^n}, a \neq 0$
When <i>multiplying</i> two expressions with the same base, we <i>add</i> the exponents.	$a^m a^n = a^{m+n}$
When <i>dividing</i> two expressions with the same base, we <i>subtract</i> the exponents.	$\frac{a^m}{a^n} = a^{m-n}$
When raising an expression with an exponent to another exponent, we multiply the exponents.	$(a^m)^n = a^{mn}$

Section 1.5 Exercises

Simplify, assuming no division by zero.

- a^0 , where a is a positive number
- $(a + b + c + d)^0$, where $a + b + c + d \neq 0$
- $2^0 - 3^0$
- $5^{-1} - 6^{-1}$

5. $x^0 + \left(\frac{1}{4}\right)^{-1}$

6. $5\left(w^0 - \left(\frac{1}{5}\right)^{-1}\right)$

7. $5^{-2} - 7^{-2}$

8. $\left(\frac{3}{7}\right)^{-2} + \left(\frac{9}{2}\right)^{-2}$

9. $4\left(\frac{3}{2}\right)^{-3} - \left(\frac{2}{x}\right)^{-2}$

10. x^2x^{18}

11. $x^2(x + 2y)$

12. $3x(4 - 7y - x^{-1})$

13. $2xy(-3x^2y^4)$

14. $x^2[x - y(3 - x)]$

15. $x^3[4 - x^2(5 - x) + 7]$

16. $\frac{1}{2}a^2(ab - a^3)$

17. $x^{-2}\left(5 - \frac{x^3}{x^2}\right)$

18. $\frac{x^6}{x^{63}}$

19. $\frac{x^{-5}y^{-2}}{y^{-8}x^{-3}}$

20. $\frac{x^2(x^{-3}x)}{y^2y^{-2}}$

21. $(x^2)^3$

22. $(x^2)^{-3}$

23. $(x^{-2})^{-3}$

24. $\frac{(x^{-2})^{-5}}{(x^2)^3}$

$$25. \frac{(2x^2yz^4)^3}{(4x^{-3}y^2z^5)^2}$$

$$26. \frac{(9w^{-3}z^{-4})^{-2}}{(3w^{-2}z^{-5})^{-3}}$$

1.6: USING FORMULAS

In this section we will be evaluating formulas where the variables are taken to be specific numerical values. First up, we need to discuss evaluating expressions.

Evaluating Expressions

An expression allows variables to be anything within reason. So when evaluating expressions, the most important thing to remember is to *use parentheses when plugging in the value*.

Example: Evaluate x^2 when $x = 2$.

Solution: We have $(2)^2 = 4$.

It seems easy here, but the next example shows why we need the parentheses.

Example: Evaluate $-x^2$ in these instances: (a) when $x = 2$ and (b) when $x = -2$.

Solution:

(a) Remember that $-x^2$ is the opposite of the square of x . We need parentheses around the variables we are substituting. So $-(2)^2 = -4$.

(b) When $x = -2$,

$$-(-2)^2 = -(4) = -4.$$

Example: Evaluate $-x^2 - x + \frac{2}{7}$ when $x = -1$.

Solution: We have

$$\begin{aligned} -(-1)^2 - (-1) + \frac{2}{7} &= -(1) + 1 + \frac{2}{7} \\ &= 0 + \frac{2}{7} \\ &= \frac{2}{7}. \end{aligned}$$

Example: Evaluate $\frac{2}{x^3} - x^2$ when $x = -4$.

Solution: We substitute in using parentheses, and obtain

$$\begin{aligned} \frac{2}{(-4)^3} - (-4)^2 &= \frac{2}{-64} - 16 \\ &= -\frac{1}{32} - 16 \\ &= -\frac{1}{32} - \frac{512}{32} \\ &= -\frac{513}{32}. \end{aligned}$$

Note how we moved the negative sign out front when we went from the first line to the second line in our solution.

Formulas

Now that we can handle expressions, formulas are just an extension of what we already know. A formula has an equal sign in it. Some examples are $A = s^2$ or $d = rt$. It's important to note the difference between an expression and a formula because we have different rules for expressions and equations. Expressions do not have an equal sign, whereas formulas and equations do. (We will discuss this difference more in future sections.)

The formula $d = rt$ is the formula that relates “distance d traveled at a constant rate r over a time period t .”

Ex) Suppose we travel for four hours at a constant speed of 50 miles per hour. Then the time $t = 4$ hours, and the rate $r = 50$ mph. Using the formula, the distance we've traveled is

$$d = rt = (4 \text{ hr.}) \left(50 \frac{\text{mi.}}{\text{hr.}} \right) = 200 \text{ mi.}$$

Notice the canceling of the units of “hours” in the previous example, so that only “miles” is left behind. We can use units to help us make sense about what a variable's value should be.

The formula $A = s^2$ gives the area A of a square with side length s .

Example: Find the area of a square with side length 3.5 feet.

Solution: Since the side length is $s = 3.5$ feet, we have

$$\begin{aligned} A &= (3.5 \text{ ft.})^2 \\ &= 3.5^2 \text{ ft.}^2 \\ &= 12.25 \text{ ft.}^2 \end{aligned}$$

It should make sense that an *area* is in square feet, where as a side *length* is in feet.

An exponent is sometimes called a **superscript**. If we write a number below and to the right of a variable, it is called a **subscript**. This practice is helpful in naming similar variables. For example, a_1 is read as “a one” or “a sub one.”

The total resistance R_T of two resistors R_1 and R_2 connected in parallel is given by the formula

$$R_T = \left(\frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} .$$

Example: Two resistors R_1 and R_2 are connected in parallel. If resistor R_1 has 3 ohms of resistance, and resistor R_2 has 4 ohms of resistance, what is the total resistance?

Solution: We have $R_1 = 3$, and $R_2 = 4$, so the total resistance is

$$\begin{aligned} R_T &= \left(\frac{1}{3} + \frac{1}{4} \right)^{-1} \\ &= \left(\frac{4}{12} + \frac{3}{12} \right)^{-1} \\ &= \left(\frac{7}{12} \right)^{-1} \\ &= \frac{12}{7}. \end{aligned}$$

The total resistance is $\frac{12}{7}$ ohms.

The average or mean \bar{x} (read “ x bar”) of n numbers x_1, x_2, \dots, x_n is given by

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}.$$

This means we take the sum of the numbers, and then divide that by how many numbers we have.

Example: Find the average of 3, -6 , 15, and 7.

Solution: There are four numbers here, so $n = 4$. The average is

$$\begin{aligned} \bar{x} &= \frac{3 + (-6) + 15 + 7}{4} \\ &= \frac{19}{4}, \end{aligned}$$

or 4.75.

Section 1.6 Exercises

- Evaluate $\frac{1}{x} + 4x$ when $x = 2$.
- Evaluate $-x^2 - 18x - 15$ when $x = -1$.
- Evaluate $\frac{x^2y^3}{x^2+y^3}$ when $x = 1$ and $y = 4$.
- Evaluate $\frac{x^2y^3}{x^2+y^3}$ when $x = -1$ and $y = -2$.
- Evaluate $y(3 - x^2)$ when $x = -2$ and $y = -3$.
- An equilateral triangle is a triangle whose side lengths are all the same. The area of an equilateral triangle with side length s is $A = \frac{\sqrt{3}}{4}s^2$.
 - Find the area of an equilateral triangle with side length 2 cm.
 - Find the area of an equilateral triangle with side length $\frac{1}{2}$ cm.
- Use the formula for total resistance to find the total resistance when $R_1 = 5$ ohms and $R_2 = 8$ ohms.
- Use the formula for total resistance to find the total resistance when $R_1 = \frac{1}{4}$ and $R_2 = \frac{1}{2}$ ohms.
- Use the formula for the mean to find the average of 1, -15, 32, 20, and -10.
- Use the formula for the mean to find the average of -3, -6, 19, and 10.
- Looking ahead: The *discriminant* is used to describe the solutions of quadratic equations. The discriminant of the quadratic $ax^2 + bx + c$ is $b^2 - 4ac$.
 - Calculate the discriminant of $2x^2 - x + 1$.
 - Calculate the discriminant of $\frac{1}{2}x^2 + \frac{1}{2}x - 7$.
- Looking ahead: The kinetic energy of an object in motion is $K = \frac{1}{2}mv^2$, where m is the object's mass and v is the object's velocity. Find the kinetic energy of a 5-kilogram object moving at $15 \frac{\text{m}}{\text{sec}}$. The units for kinetic energy are **joules**, which are kilograms times meters² divided by seconds², or $\text{J} = \text{kg} \cdot \frac{\text{m}^2}{\text{sec}^2}$.

CHAPTER 1 REVIEW

1-A

1. Of real number, rational number, irrational number, integer, and natural number, which of these can be applied to $\sqrt{2}$?
2. Of real number, rational number, irrational number, integer, and natural number, which of these can be applied to $1.\overline{7}$?
3. Simplify $14 + 1 \cdot 2$.
4. Simplify $2^2 + 3^2$.
5. Simplify $x^2 + 3x + 5x^2 + 6x$.
6. Distribute to simplify $2(x - 3)$.
7. Simplify x^2x^3 .
8. Evaluate $x^2 + x + 1$ when $x = -1$.

1-B

1. Of real number, rational number, irrational number, integer, and natural number, which of these can be applied to π ?
2. Of real number, rational number, irrational number, integer, and natural number, which of these can be applied to $\pi - \pi$?
3. Simplify $14 - 1 \cdot 2 + 4 \div 3$.
4. Simplify $\left(-\frac{2}{3}\right)^2 + \left(-\frac{3}{4}\right)^2$.
5. Simplify $x^2y^2 + 4xy^2 + 8x^2y - 10xy^2 - 7x^2y + 3x^2y^2$.
6. Simplify $2[3 - (5 - x)]$.
7. Simplify $\frac{x^4}{y^7} \cdot \frac{y^3}{x^{-6}}$.
8. Evaluate $x^2y + xy^2 + x + y$ when $x = -1$ and $y = -1$.