

FIRST EDITION

Handbook for Undergraduate

# GENERAL PHYSICS



ANDREW CARMICHAEL

# Handbook for Undergraduate General Physics

*First Edition*

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# Contents

|  |           |
|--|-----------|
| <b>Introduction</b>                                  | <b>v</b>  |
| <b>I Physics</b>                                     | <b>1</b>  |
| <b>1 Mechanics</b>                                   | <b>3</b>  |
| 1.1 Kinematics, Dynamics and Gravitation . . . . .   | 3         |
| 1.2 Elasticity and Simple Harmonic Motion . . . . .  | 10        |
| 1.3 Fluids and Flow Equations . . . . .              | 11        |
| <b>2 Electricity and Magnetism</b>                   | <b>13</b> |
| 2.1 Electric Force, Field, and Potential . . . . .   | 13        |
| 2.2 Magnetic Force and Field . . . . .               | 18        |
| 2.3 Circuits . . . . .                               | 21        |
| 2.4 Electromagnetic Waves (EM) . . . . .             | 26        |
| 2.5 Numerical Data . . . . .                         | 27        |
| <b>3 Optics</b>                                      | <b>29</b> |
| 3.1 Geometrical Optics and General Results . . . . . | 29        |
| 3.2 Diffraction and Interference . . . . .           | 30        |
| 3.3 Mirrors and Lenses . . . . .                     | 32        |

|            |                            |           |
|------------|----------------------------|-----------|
| <b>4</b>   | <b>Mechanical Waves</b>    | <b>33</b> |
| 4.1        | Mechanical Waves . . . . . | 33        |
| <b>5</b>   | <b>Thermal Physics</b>     | <b>35</b> |
| 5.1        | Thermal Physics . . . . .  | 35        |
| 5.2        | Thermodynamics . . . . .   | 37        |
| 5.3        | Numerical Data . . . . .   | 37        |
| <b>6</b>   | <b>Modern Physics</b>      | <b>39</b> |
| <b>II</b>  | <b>Mathematics</b>         | <b>41</b> |
| <b>7</b>   | <b>Math Formulae</b>       | <b>43</b> |
| <b>III</b> | <b>Tables</b>              | <b>51</b> |

# Introduction

## Purpose

The purpose of this handbook is to fill a gap in the current market of physics reference books for students. Several (often quite good) handbooks exist pitched at an advanced level, useful to physics majors, graduate students, and researchers. Conversely, study guides and laminated formula sheets existing at approximately the high school level are abundant. The present book is in between the two, primarily aimed at students taking usually a two or three semester physics sequence at college level pursuant to paths in either the life sciences or engineering. Such courses are often split into calculus and non-calculus based versions. The present book includes some calculus but, hopefully, not too intrusively and will hopefully be useful to students taking either. It may furthermore be useful to physics majors as a reference.

## Notational Niceties

Each passing edition of textbooks at this level yields somewhat of a convergence of notation. However, the limited number of letters in both the Latin and Greek alphabets coupled with tradition and the fact that various texts disagree still means ambiguities in notation will always exist. This section is designed to provide some clarification.

## Mechanics

- In mechanics, upper case  $T$  often refers to tension but also to time period. This can cause confusion in pendulum problems.
- Torque is often denoted either by (an enlarged)  $\tau$  or  $\Gamma$ . This book prefers  $\Gamma$  (as it resembles a handle).
- Different books use either  $T$  or  $K$  for kinetic energy. We use  $K$ . Similarly, some use  $V$  and some use  $U$  for potential energy. We use  $U$  for potential energy and  $V$  for potential.

## Electromagnetism

- The symbol  $V$  commonly refers to electric potential. A couple of points should be made:

- Some texts use  $\Phi$  for potential. We use  $V$  for potential and  $\Phi$  for flux.
- The term voltage refers to (usually) the absolute value of the potential difference (PD or p.d.) between two points either in space or in a circuit. Often for brevity we replace  $|\Delta V| \rightarrow V$ .
- The unit for either potential or voltage is the volt, symbol V (roman font).

## Thermodynamics

- In thermodynamics there is a general tendency to use uppercase letters for extensive quantities (such as  $V$ , entropy  $S$ , internal energy  $U$ , any of the thermodynamic potentials such as enthalpy  $H$ ) and lowercase letters for intensive quantities (such as density  $\rho$ , pressure  $p$ ). This cannot always be adhered to; for example temperature is intensive but denoted by  $T$ .
- Heat capacities are usually total (extensive), molar (per mole; intensive) or specific (per unit mass; intensive). This book uses uppercase  $C$  for the extensive total heat capacity (also often called thermal mass) and lowercase  $c$  for any intensive version. Which intensive version is meant should be clear from units where numerical values are given.
- In thermodynamics, by tradition  $U$  denotes the total internal energy of a system, such as a gas. For an ideal gas, this is the sum total of the kinetic energies of its constituent particles. Since the ideal gas neglects interactions, no potential energy exists. Possible confusion exists here because in mechanics,  $U$  denotes potential energy only.

**Part I**

**Physics**

# Chapter 1

## Mechanics

### 1.1 Kinematics, Dynamics and Gravitation

#### 1D Motion

##### Position, Velocity and Acceleration

$$\Delta x / \Delta t = v_{av}$$

$$dx/dt = v(t)$$

$$\int a(t)dt = \Delta v$$

$$\Delta v / \Delta t = a_{av}$$

$$dv/dt = a(t)$$

$$\int v(t)dt = \Delta x$$

##### Uniformly Accelerated Linear Motion ( $a=\text{const.}$ )

$$v = v_0 + at$$

$$v^2 = v_0^2 + 2as$$

$$s = v_0t + \frac{1}{2}at^2$$

$$s = \frac{v_0 + v}{2}t$$

#### 2D Motion

##### 2D Motion: Projectile Motion

##### Projectile Motion 2D (uniform field $g=\text{const.}$ )

Initial Condition:

$$x'(0) = \dot{x}(0) = v_x(0) = v(0) \cos \theta = v_0 \cos \theta$$

$$y'(0) = \dot{y}(0) = v_y(0) = v(0) \sin \theta = v_0 \sin \theta$$

$$y'' = -g = \text{const.}$$

$$y'(t) = y'(0) - gt$$

$$y(t) = y(0) + y'(0)t - \frac{1}{2}gt^2$$

$$x'' = 0 = \text{const.}$$

$$x'(t) = x'(0) = \text{const.}$$

$$x(t) = x(0) + x'(0)t$$

**Alternative Form**

$$\begin{array}{ll}
 a_y = -g = \text{const.} & a_x = 0 = \text{const.} \\
 v_y(t) = v_y(0) - gt & v_x(t) = v_x(0) = \text{const.} \\
 y(t) = y(0) + v_y(0)t - \frac{1}{2}gt^2 & x(t) = x(0) + v_x(0)t
 \end{array}$$

**Trajectory Equation** (in terms of  $v(0)$ ,  $g$ )

$$y(x) = - \left[ \frac{1}{2} \frac{g}{v_x^2(0)} \right] x^2 + \left[ \frac{v_y(0)}{v_x(0)} \right] x + y(0)$$

**Velocity Position Equations**

$$\begin{aligned}
 v^2(y) &= v^2(0) - 2gy \\
 v_y^2(y) &= v_y^2(0) - 2gy
 \end{aligned}$$

**Special Case: Level Ground****Special Points (for Level Ground): Range, Height, Flight time**

|                |     |     |                   |     |                             |
|----------------|-----|-----|-------------------|-----|-----------------------------|
| height         | $h$ | $=$ | $v_y^2(0)/2g$     | $=$ | $v^2(0) \sin^2 \theta / 2g$ |
| Range          | $R$ | $=$ | $2v_x(0)v_y(0)/g$ | $=$ | $v^2(0) \sin 2\theta / g$   |
| Time of flight | $T$ | $=$ | $2v_y(0)/g$       | $=$ | $2v(0) \sin \theta / g$     |

$$R = 4h \cot \theta$$

**Trajectory Equation** (with initial condition  $y(0) = 0$ )

$$\begin{aligned}
 y(x) &= - \left[ \frac{\tan \theta}{R} \right] x^2 + x \tan \theta \\
 y(x) &= x(R - x) \left[ \frac{\tan \theta}{R} \right]
 \end{aligned}$$

**Energy for Projectile Motion**

|                               |  |
|-------------------------------|--|
| GPE uniform field             | $U(y) = mgy + U(0)$                          |
| Potential uniform grav. field | $V(y) = gy + V(0)$                           |
| Total energy                  | $E = \frac{1}{2}mv^2 + mgy$                  |
| GPE uniform field             | $\Delta U_{\text{grav.}} = mg\Delta y = mgh$ |

Note: We can always choose  $U(0) = 0$ ,  $V(0) = 0$ .

**2D Motion: Circular Motion**

|                           |                            |
|---------------------------|----------------------------|
| Centripetal acceleration  | $a_r = v^2/r = r\omega^2$  |
| Arc length                | $s = r\theta$              |
| Tangential speed          | $v = r\omega = 2\pi r/T$   |
| Tangential acceleration   | $a_t = r\alpha$            |
| Angular frequency         | $\omega = 2\pi f = 2\pi/T$ |
| Frequency and time period | $f = 1/T$                  |
| Uniform circular motion   | $a_t = 0, \alpha = 0$      |

**Forces and Momentum**

|                                 |   |
|---------------------------------|---|
| Newton's second law (general)   | $\vec{F} = d\vec{p}/dt$                     |
| Potential energy and force (1D) | $F = -dU/dx$                                |
| Potential energy and force (3D) | $\vec{F} = -\nabla U$                       |
| Linear momentum                 | $\vec{p} = m\vec{v}$                        |
| Impulse for constant force      | $\vec{J} = \Delta\vec{p} = \vec{F}\Delta t$ |
| Friction (static)               | $f_s \leq f_{s,\max} = \mu_s N$             |
| Friction (kinetic)              | $f_k = \mu_k N$                             |

**Gravitation and Orbits Gravitational Fields Due to Point or Spherical Sources**

|                           |                |
|---------------------------|----------------|
| Force between masses      | $F = Gmm'/r^2$ |
| Gravity field of mass $m$ | $g = Gm/r^2$   |
| G.P.E. two masses         | $U = -Gmm'/r$  |
| Grav. potential of $m$    | $V = -Gm/r$    |

**Orbital Motion**

|                              |  |
|------------------------------|--|
| Kepler's 2 <sup>nd</sup> Law | $T^2 = (4\pi^2/GM)r^3$                 |
| Orbit (circular)             | $v^2 = GM/r$                           |
| Escape velocity              | $v^2 = 2GM/r$                          |
| Orbital energy               | $E = \frac{1}{2}mv^2 - \frac{Gmm'}{r}$ |
| Rocket equation              | $a = \frac{v_e}{m} \frac{dm}{dt} - g$  |
| Rocket equation              | $\Delta v = v_e \ln \frac{m_i}{m_f}$   |

**Constants Related to Gravity**

|                                 |   |
|---------------------------------|---|
| Universal const. of gravitation | $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ |
| Earth surface gravity           | $g = 9.81 \text{ m/s}^2$  |
| Earth mass & $G$                | $GM_E = 3.98 \times 10^{14} \text{ m}^3/\text{s}^2$               |
| Solar mass & $G$                | $GM_\odot = 1.33 \times 10^{20} \text{ m}^3/\text{s}^2$           |
| Moon mass & $G$                 | $GM_\zeta = 4.91 \times 10^{12} \text{ m}^3/\text{s}^2$           |

**Work and Energy**

|                               |   |
|-------------------------------|---|
| Kinetic energy                | $K = \frac{1}{2}mv^2$                             |
| Work Done by Force $\vec{F}$  | $W = \vec{F} \cdot \vec{s} = Fs \cos \theta$      |
| Work done, const. force       | $W = Fs \cos \varphi = F_{\parallel} \cdot s$     |
| Work                          | $W = \int \vec{F} \cdot d\vec{r}$                 |
| Power                         | $P = dE/dt = dW/dt$                               |
| Average power                 | $P_{\text{av}} = \Delta E/\Delta t = W/\Delta t$  |
| Instantaneous power           | $P = \vec{F} \cdot \vec{v} = F_{\parallel}v$      |
| Work-energy theorem           | $W_{\text{net}} = W_c + W_{\text{nc}} = \Delta K$ |
| Work done by con. forces      | $W_c = -\Delta U$                                 |
| Mechanical energy             | $E_{\text{mech}} = K + U$                         |
| Conservation of mech. energy  | $K_i + U_i + W_{\text{nc}} = K_f + U_f$           |
| Work done by non-con. forces  | $W_{\text{nc}} = -\Delta E_{\text{mech}}$         |
| GPE uniform field             | $U(y) = mgy + U(0)$                               |
| Potential uniform grav. field | $V(y) = gy + V(0)$                                |
| GPE uniform field             | $\Delta U_{\text{grav.}} = mg\Delta y = mgh$      |
| Spring potential energy       | $U = \frac{1}{2}kx^2$                             |

**Center of Mass**

$$\vec{R}_{\text{cm}} = \frac{1}{M} \sum m_i \vec{r}_i \qquad \vec{R}_{\text{cm}} = \frac{1}{M} \int \vec{r} dm = \frac{1}{M} \int \vec{r} \rho dV$$

**Theorems for Variable Forces**

|                  |   |
|------------------|---|
| Impulse-momentum | $\vec{J} = \Delta \vec{p} = \vec{F}_{\text{av}} \Delta t = \int \vec{F}_{\text{net}}(t) dt$ |
| Work-energy      | $W_{\text{net}} = \int \vec{F}_{\text{net}} \cdot d\vec{r} = \Delta K$                      |

## Collisions

### Types of Collision

- Totally elastic: no loss of K.E.
- Inelastic: some loss of K.E.
- Completely inelastic:  $v_1 = v_2 = v$ , maximum K.E. loss

### Conservation Laws

Momentum conservation

$$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$

K.E. conservation (**elastic case only**)

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Newton's collision law (**1D collisions only**)

$$(v_2 - v_1) = -e(u_2 - u_1)$$

### Newton's Law of Collisions (1D case only)

$e$  = Newton's coefficient of restitution

- totally elastic: No loss of K.E.,  $e = 1$
- inelastic: Some loss of K.E.,  $0 < e < 1$
- completely inelastic:  $v_1 = v_2 = v$ ,  $e = 0$  Max K.E. loss

**1D case** combining  $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$  with  $(v_2 - v_1) = -e(u_2 - u_1) \implies$

$$v_1 = \frac{m_1 - em_2}{m_1 + m_2}u_1 + \frac{(1+e)m_2}{m_1 + m_2}u_2$$

$$v_2 = \frac{(1+e)m_1}{m_1 + m_2}u_1 + \frac{m_2 - em_1}{m_1 + m_2}u_2$$

## Rotational Motion

### Fundamentals

|                                       |  |
|---------------------------------------|--|
| Angular velocity, acceleration        | $\omega = d\theta/dt, \alpha = d\omega/dt$                           |
| Angular displacement                  | $\Delta\theta = \int \omega dt$                                      |
| Linear and angular connection         | $v_t = R\omega, a_t = R\alpha$                                       |
| Torque                                | $\vec{\Gamma} = \vec{r} \times \vec{F}$                              |
| Magnitude of torque                   | $\Gamma = rF \sin \varphi = rF_{\perp}$                              |
| Angular momentum (particle)           | $\vec{L} = \vec{r} \times \vec{p}$                                   |
| Angular momentum (solid)              | $\vec{L} = I\vec{\omega}$  |
| Moment of inertia (particles)         | $I = \sum mr_{\text{axis}}^2$  |
| Moment of inertia (solid)             | $I = \int r_{\text{axis}}^2 dm$                                      |
| N2 for rotation (general form)        | $\vec{\Gamma} = d\vec{L}/dt$   |
| N2 for rotation ( $I=\text{const.}$ ) | $\vec{\Gamma} = I\vec{\alpha}$                                       |
| Rotational K.E.                       | $K_r = \frac{1}{2}I\omega^2$   |
| Work done by a torque                 | $W = \int \Gamma \cdot d\theta = \Delta K_r$                         |
| Rotational power                      | $P = \Gamma\omega$   |
| Conservation of $\vec{L}$             | $I_i\omega_i = I_f\omega_f$  |
| Rolling without slipping              | $v_{\text{cm}} = R\omega, a_{\text{cm}} = R\alpha$                   |
| Parallel axis theorem                 | $I = I_{\text{cm}} + MD^2$   |
| Total kinetic energy                  | $I = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2$ |

### Uniform Rotational Motion

$$\begin{aligned} \omega &= \omega_0 + \alpha t & \Delta\theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\Delta\theta & \Delta\theta &= \frac{\omega_0 + \omega}{2}t \end{aligned}$$

### Substitutions for Rotational Dynamics

$$\begin{aligned} s &\implies \Delta\theta & \vec{F} &\implies \vec{\Gamma} \\ u &\implies \omega_0 & m &\implies I \\ v &\implies \omega & K &= \frac{1}{2}mv^2 \implies K_r = \frac{1}{2}I\omega^2 \\ a &\implies \alpha & \vec{p} = m\vec{v} &\implies \vec{L} = I\vec{\omega} \end{aligned}$$

**Moments of Inertia**

| Moment                            | Object                | Axis              |
|-----------------------------------|-----------------------|-------------------|
| $I = MR^2$                        | Uniform ring/tube     | Through C.M.      |
| $I = \frac{1}{2}MR^2$             | Uniform disk/cylinder | Through C.M.      |
| $I = \frac{1}{2}M(R_1^2 + R_2^2)$ | Annular cylinder      | Through C.M.      |
| $I = \frac{1}{12}ML^2$            | Uniform rod           | Through C.M.      |
| $I = \frac{1}{3}ML^2$             | Uniform rod           | Through end       |
| $I = \frac{2}{5}MR^2$             | Uniform sphere        | Through C.M.      |
| $I = \frac{2}{3}MR^2$             | Hollow sphere         | Through C.M.      |
| $I = \frac{1}{3}Ma^2$             | Slab width $a$        | Along edge (door) |

## 1.2 Elasticity and Simple Harmonic Motion

### Simple Harmonic Motion (SHM)

|                              |  |
|------------------------------|--|
| Hooke's Law                  | $F(x) = -kx$   |
| Acceleration                 | $a(x) = -\omega^2 x = -n^2 x$                                      |
| Velocity                     | $v(x) = \pm \omega \sqrt{A^2 - x^2}$                               |
| SPE or EPE for a spring      | $U(x) = \frac{1}{2} k x^2 = \frac{1}{2} m \omega^2 x^2$            |
| Total energy                 | $E = \frac{1}{2} k A^2 = \frac{1}{2} m \omega^2 A^2$               |
| Position $x(t)$              | $x(t) = A \sin(\omega t + \varphi)$                                |
| Velocity $v(t)$              | $v(t) = A \omega \cos(\omega t + \varphi)$                         |
| Acceleration $a(t)$          | $a(t) = -A \omega^2 \sin(\omega t + \varphi)$                      |
| Constant of motion $\varphi$ | $\tan \varphi = \frac{\omega x(0)}{v(0)}$                          |
| Constant of motion $A$       | $A^2 = x^2(0) + \frac{v^2(0)}{\omega^2}$                           |
| Period, mass-spring          | $T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$  |
| Period, simple pendulum      | $T = \frac{1}{f} = \frac{2\pi}{n} = 2\pi \sqrt{\frac{l}{g}}$       |
| Period, physical pendulum    | $T = \frac{1}{f} = \frac{2\pi}{n} = 2\pi \sqrt{\frac{I}{mgr}}$     |
| Period, torsional pendulum   | $T = \frac{1}{f} = \frac{2\pi}{n} = 2\pi \sqrt{\frac{I}{\kappa}}$  |
| Period, floating object      | $T = \frac{1}{f} = \frac{2\pi}{n} = 2\pi \sqrt{\frac{m}{g\rho A}}$ |

### Elasticity (Cables) ( $\delta l = x = \text{extension}$ )

|                              |  |
|------------------------------|--|
| Tensile stress               | stress = $F/A$   |
| Tensile strain               | strain = $\delta l/l_0$  |
| Young's modulus              | $Y = \frac{\text{stress}}{\text{strain}} = \frac{F/A}{\delta l/l_0}$               |
| Young's modulus (diff. form) | $Y = \frac{dF/A}{dl/l_0} = \frac{l_0}{A} \frac{dF}{dl}$                            |
| Hooke's law (diff. form)     | $dF/dl = (YA/l_0)$   |
| Hooke's law (int. form)      | $F = (YA/l_0) \cdot \delta l = (YA/l_0) \cdot x = k \cdot x$                       |
| Strain energy                | $W = U = \frac{1}{2} F \cdot \delta l = \frac{1}{2} F \cdot x = \frac{1}{2} k x^2$ |

Note:  $\Delta l$  and  $\delta l$  can both represent change in length (extension). The only difference being

that  $\delta$  refers to a small change,  $\Delta$  represents any change. Either is acceptable within the Hooke limit.

### Elastic Response Functions

|                           |   |
|---------------------------|---|
| Hooke's law (diff. form)  | $Y = \frac{dF/A}{dl/l_0} = \frac{l_0}{A} \frac{dF}{dl}$ |
| Hooke's law (int. form)   | $\Delta l = l - l_0 = \left[ \frac{l_0}{YA} \right] F$  |
| Hooke's law (int. form)   | $l = l_0 \left[ 1 + \frac{1}{YA} F \right]$             |
| Compressibility           | $\beta = -\frac{1}{V} \frac{dV}{dp}$                    |
| Bulk modulus (diff. form) | $B = \frac{1}{\beta} = -V \frac{dp}{dV}$                |
| Bulk modulus (int. form)  | $\Delta V = V - V_0 = -\frac{V_0}{B} p$                 |
| Bulk modulus (int. form)  | $V = V_0 \left[ 1 - \frac{1}{B} p \right]$              |

Note: For cables  $F = 0 \implies l = l_0$ , for bulk objects  $p = 0 \implies V = V_0$

## 1.3 Fluids and Flow Equations

### Fluids: Statics

|                              |  |
|------------------------------|--|
| Definition of pressure       | $p = F/A$  |
| Pressure with depth          | $\Delta p = h\rho g$   |
| Pressure with depth (up +ve) | $p(z) = -\rho g z + p(0)$  |
| Mass element                 | $dm = \rho dV$   |
| Archimedes' principle        | $F_b = \rho_{\text{fluid}} g V_{\text{disp.}}$                                 |
| Floating object              | $V_{\text{sub.}}/V_{\text{object}} = \rho_{\text{object}}/\rho_{\text{fluid}}$ |
| Submerged object             | $s = W/\Delta W = \rho_{\text{object}}/\rho_{\text{fluid}}$                    |

### Fluids: Dynamics

|                         |   |
|-------------------------|---|
| Continuity equation     | $A_1 v_1 = A_2 v_2$   |
| Bernoulli's equation    | $p + \frac{1}{2} \rho v^2 + \rho g y = \text{const.}$                       |
| Poiseuille Law (volume) | $\frac{dV}{dt} = - \left[ \frac{\pi r^4}{8\eta} \right] \frac{dp}{dx}$      |
| Poiseuille Law (mass)   | $\frac{dm}{dt} = - \left[ \frac{\pi r^4 \rho}{8\eta} \right] \frac{dp}{dx}$ |

**Flow Equations Summary**

|              | Differential form   |
|--------------|---|
| Fluid (vol.) | $\frac{dV}{dt} = - \left[ \frac{\pi r^4}{8\eta} \right] \frac{dp}{dx}$    |
| Fluid (mass) | $\frac{dm}{dt} = - \left[ \frac{\rho r^2}{8\eta} \right] A \frac{dp}{dx}$ |
| Electric     | $\frac{dq}{dt} = I = - \frac{A}{\rho} \frac{dV}{dx}$                      |
| Heat         | $\frac{dQ}{dt} = -kA \frac{dT}{dx}$                                       |